THEORY OF SHOCK WAVES IN MEDIA UNDERGOING PHASE TRANSITIONS

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It is well known that many solids can exist in different crystalline modifications under diverse conditions. For certain temperature and pressure values, joined by a definite interdependence, transitions can take place from one crystalline modification to another. These transitions, which are accompanied by volume shock and release (absorption) of latent heat, are called first-order phase transitions. First-order phase transitions often occur at high pressures. The present study comprises a theoretical analysis of certain laws of shock propagation in solids undergoing first-order phase transitions. We investigate shock waves in the case of moderate pressures, so that the entropy growth is small and the shock adiabat is close to isentropic.

It is known that a shock having an amplitude of even 100 kbar in a solid is still weak. Such a shock differs only slightly from a sound wave, because it propagates at close to the sonic velocity and imparts to the medium behind the front just one-tenth the value of the wave propagation velocity itself.

At the same time, the pressure in the shock must be large enough to render strength effects negligible and to permit the shock wave to be treated as a plastic phenomenon (the tensile strength is normally ~1 kbar).

§ 1. Fundamental Equations

We consider the pressure to be hydrostatic and investigate a plane one-dimensional mathematical model. The system of equations describing propagation of a finite-amplitude wave under conditions of first-order phase transitions has the form

$$\begin{gathered}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x},\ \rho=\frac{\rho_1}{1-\alpha\left(\frac{V_1-V_2}{V_1}\right)},\\ \frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}=0,\ \delta p=c_\infty^2\delta\rho-(c_\infty^2-c_0^2)\int\limits_0^\infty\delta\rho(t-t')\,\varphi\left(\frac{t'}{\theta}\right)d\frac{t'}{\theta},\\ \delta\rho_1=\frac{1}{c_1^2(p)}\delta p,\ \delta\rho_2=\frac{1}{c_2^2(p)}\delta p,\end{gathered}\tag{1.1}$$

where u is the particle velocity in the wave; ρ is the density of the mixture of both phases; α is the mass fraction of the second phase in the system; ρ_1, ρ_2, c_1, and c_2 are the densities and sound velocities in the first and second phases, respectively; and $V_1=1/\rho_1$, $V_2=1/\rho_2$ are the specific volumes of the first and second phases.

The fourth equation in the system (1.1) characterizes the relationship between $\delta\rho$ and δp in the case where the density variation is accompanied by a relaxation process [1]. Here, of course, it is necessary to take account of the dependence of δp on the density variations at preceding times.

The function $\varphi(t/\theta)$, describing the relaxation process vanishes rapidly for $t>\theta$, where θ is the effective relaxation time (in the given case θ is the characteristic interphase transition time).

For definiteness we normalize the indicated function

$$\int\limits_0^\infty\varphi\left(\frac{t}{\theta}\right)d\frac{t}{\theta}=1.$$

The high-frequency sound velocity $c_\infty=c_{\omega\theta\to\infty}=c_\alpha$ is obtained from the third equation of the system (1.1)

$$\frac{1}{c_\infty^2}=\frac{V_1^2}{V_2}(1-\alpha)\frac{1}{c_1^2}+\frac{V_2^2}{V^2}\alpha\frac{1}{c_2^2}.$$

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It has been shown [2] that the low-frequency sound velocity $c_0^2 = (\partial p/\partial \rho)_{\omega\theta\to 0}$ for first-order phase transition in the case of zero shear modulus is given by the expression

$$\frac{1}{c_0^2} = \rho^2\left\{\alpha\left[-\left(\frac{\partial V_2}{\partial p}\right)_T - \frac{2T}{q}\left(\frac{\partial V_2}{\partial T}\right)_p (V_2 - V_1) + \frac{Tc_{p2}}{q^2}(V_2 - V_1)^2\right] + \right.$$
$$\left. + (1-\alpha)\left[-\left(\frac{\partial V_1}{\partial p}\right)_T - \frac{2T}{q}\left(\frac{\partial V_1}{\partial T}\right)_p (V_2 - V_1) + \frac{Tc_{p1}}{q^2}(V_2 - V_1)^2\right]\right\},$$

in which c_{p1}, c_{p2} are the specific heats of the first and second phases at constant pressure, q is the latent heat of transition, and T is the temperature.

If the shock wave propagates without phase transition, then $\alpha = 0$ and shock propagation is described by the first, second, and fifth equations of the system (1.1).

§ 2. Propagation Velocity and Profile of Shock Waves with Phase-Transition Relaxation and Incomplete Conversion to the Second Phase

We consider the system (1.1) in the case of shock propagation under conditions of phase transition. Polymorphic conversion normally takes place in a time much greater than the time required for the settling of thermodynamic equilibrium in an ordinary single-phase medium. The width of the shock front in the presence of phase transition is determined in this case by the effective transition relaxation time. We analyze the structure of a plane shock wave with phase-transition relaxation. We adopt, as usual [1], the functions $u(x - wt)$, $p(x - wt)$, $\rho(x - wt)$, where w is the shock velocity.

From the first two equations of the system (1.1) we obtain

$$u = -\frac{w\rho_0}{\rho} + w,\; p = \rho_0 w^2 - \frac{\rho_0^2 w^2}{\rho} + p_0, \tag{2.1}$$

where ρ_0 is the initial density of the two-phase mixture.

From expression (2.1) and the fourth equation of the system (1.1) we obtain

$$\rho_0 w^2 - \frac{\rho_0^2 w^2}{\rho_0 + \delta\rho} = c_\infty^2 \delta\rho - (c_\infty^2 - c_0^2)\int\limits_0^\infty \delta\rho\,(t - t')\,\varphi\left(\frac{t'}{\theta}\right)\frac{dt'}{\theta}. \tag{2.2}$$

We now consider the case in which the relaxation time is small in comparison with the characteristic wave period. Then the quantity $\delta\rho(t - t')$ in (2.2) can be expanded in powers of t'. Limiting the expansion to two terms, we have

$$\rho_0 w^2 - \frac{\rho_0^2 w^2}{\rho_0 + \delta\rho} = c_0^2\delta\rho + \xi\frac{\partial\delta\rho}{\partial t} - \beta\frac{\partial^2\delta\rho}{\partial t^2}, \tag{2.3}$$

where

$$\xi = (c_\infty^2 - c_0^2)\,\theta\int\limits_0^\infty \mu_1\varphi(\mu_1)\,d\mu_1;$$

$$\beta = \frac{1}{2}(c_\infty^2 - c_0^2)\,\theta^2\int\limits_0^\infty \mu_1^2\varphi(\mu_1)\,d\mu_1.$$

It is seen that the last term on the right-hand side of Eq. (2.3), $\beta\ddot{\delta\rho}$, is of order $\beta\delta\rho/\Xi^2$, smaller than $\xi\dot{\delta\rho} \sim \xi\delta\rho/\Xi$ and can be dropped (Ξ is the wave period). For $(\partial V_2/\partial p)_T \approx (\partial V_1/\partial p)_T$, $(\partial V_2/\partial T)_p \approx (\partial V_1/\partial T)_p$, $c_{p1} \approx c_{p2}$ the dependence of c_0 on α can be neglected.

Following [1], we assume that ξ is a first-order small quantity. We then obtain from Eq. (2.3), correct to second-order terms,

$$\mu\delta\rho' - (c_0/2\rho_0)(\delta\rho)^2 + (\Delta u/2)\delta\rho = 0, \tag{2.4}$$

where $\delta\rho' = \partial\delta\rho/\partial(x - wt)$; $\mu = \xi/2$; $\Delta u = u(x = -\infty) - u(x = \infty)$.

Equation (2.4) represents the once-differentiated stationary Burgers equation [1] with respect to the density variation.

Its solution under the condition $\delta\rho(x=\infty)=0$ has the form

$$\delta\rho=\frac{\frac{\rho_0}{c_0}\Delta u}{1+\exp\frac{\Delta ux}{2\mu}}. \tag{2.5}$$

It is known that the solution (2.5) represents a shock wave with a discontinuity of magnitude $(\rho_0/c_0)\Delta u$ and a transition zone of width $\delta=2\mu/\Delta u$, which vanishes as $\mu\to 0$. The quantity of matter of the new phase in the shock wave is determined from the fourth equation of the system (1.1). Thus, for $\alpha\ll 1$ we obtain

$$\alpha\approx\frac{V_1}{V_1-V_2}\left(1-\frac{c_0^2}{c_1^2}\right)\frac{\Delta u}{c_0}\frac{1}{1+\exp\frac{\Delta ux}{2\mu}}.$$

It is important to note that the shock structure described by Eq. (2.5) occurs at definite intensities, above which the shock structure changes considerably.

Thus, the characteristic period in the shock wave is $\Xi=\delta/w=2\mu/\Delta uw$, $w=c_0+\Delta u/2$. Expressions (2.1), (2.3), and (2.5) hold under the condition $\theta/\Xi=\theta\Delta uw/2\mu\ll 1$, whence it follows that $c_\infty^2-c_0^2\gg\Delta uw$, corresponding to the condition [2] $c_\infty>w>c_0$. In the case $c_\infty^2-c_0^2\ll\Delta uw$, we obtain from the fourth equation of the system (1.1) (1.1)

$$\delta p=c_\infty^2\delta\rho.$$

Now the medium is shock-compressed from the state ρ_{10} to the state ρ_1 without conversion to the second phase in the shock front. Expression (2.5) and the results given below are qualitatively consistent with the results of Zel'dovich and Raizer [3].

Standard methods [1] can be used to deduce the nonstationary Burgers equation in the density variation from the system of equations (1.1):

$$\frac{\partial\rho}{\partial t}+\left\{\int\frac{c_0\delta\rho}{\rho}+c_0(\rho)\right\}\frac{\partial\rho}{\partial x}=\mu\frac{\partial^2\rho}{\partial x^2}. \tag{2.6}$$

Hereinafter we neglect the compressibility of the pure phases in comparison with the compressibility of the mixture and take the dependence of c_0 on α into account. Then, substituting the expression $\rho=\frac{\rho_1}{1-\alpha\left(\frac{V_1-V_2}{V_1}\right)}$ into (2.6) and carrying out some simple transformations, we obtain the following equation in $\alpha\ll 1$:

$$\frac{\partial\alpha}{\partial t}+(c_{00}+\gamma\alpha)\frac{\partial\alpha}{\partial x}=\mu\frac{\partial^2\alpha}{\partial x^2}, \tag{2.7}$$

where

$$c_{00}=c_0(\alpha\to 0);\ \gamma=\frac{B-A}{2B\rho_1\sqrt{B}},$$

$$A=-\left(\frac{\partial V_2}{\partial p}\right)_T-\frac{2T}{q}\left(\frac{\partial V_2}{\partial T}\right)_p(V_2-V_1)+\frac{Tc_{p2}}{q^2}(V_2-V_1)^2,$$

$$B=-\left(\frac{\partial V_1}{\partial p}\right)_T-\frac{2T}{q}\left(\frac{\partial V_1}{\partial T}\right)_p(V_2-V_1)+\frac{Tc_{p1}}{q^2}(V_2-V_1)^2.$$

A nonstationary equation of the type (2.7) is known to have an exact solution satisfying the initial condition $\alpha(x,0)=\alpha_0$ and tending asymptotically to the stationary form.

We consider the stationary solution of Eq. (2.7). Let $\gamma>0$ (the second phase is less compressible). We then obtain under the condition $\alpha(x=\infty)=0$

$$\alpha=\frac{\Delta\alpha}{1+\exp\frac{\gamma\Delta\alpha x}{2\mu}},\quad w=c_{00}+\frac{\gamma\Delta\alpha}{2},\quad \Delta\alpha=\alpha(x=-\infty)-\alpha(x=\infty). \tag{2.8}$$

The solution (2.8) represents a shock formed at the leading edge of a quasisimple wave with discontinuity $\Delta\alpha$, transition zone of width $\delta=2\mu/\gamma\Delta\alpha$, and propagation velocity $w=c_{00}+\gamma\Delta\alpha/2$.

For $B<A$, i.e., for $\gamma<0$ (the second phase is more compressible), we obtain for $\alpha(x=-\infty)=0$

$$\alpha=\frac{\Delta\alpha}{1+\exp-\frac{|\gamma|\Delta\alpha x}{2\mu}},\quad w=c_{00}-\frac{|\gamma|\Delta\alpha}{2},\quad \Delta\alpha=\alpha(x=\infty)-\alpha(x=-\infty). \tag{2.9}$$

The solution (2.9) represents a shock formed at the trailing edge of a quasisimple wave, propagating with velocity $w = c_{00} - |\gamma|\Delta\alpha/2$.

For $\gamma < 0$ Eq. (2.7) is also satisfied by the expression

$$\alpha = -\frac{\Delta\alpha}{1+\exp\frac{|\gamma|\Delta\alpha x}{2\mu}}, \quad w = c_{00} + |\gamma|\Delta\alpha/2,$$

which corresponds to a rarefaction shock.

The profiles of the compression shocks (2.5), (2.8), and (2.9), in which complete conversion to the second phase does not take place, correspond to a stable double-wave structure with a shock wave present in the first phase.

§ 3. Propagation Velocity and Profile of Shock Waves with Complete Conversion to the Second Phase

Taking the example of the evolution of a compression wave, we now consider the case of complete conversion to the second phase. Let the phase-transition relaxation time be small, $\omega\theta = (2\pi/\Xi)\theta \ll 1$ (analogous results are obtained in the case of a rarefaction wave).

Let us assume that a piston is moving according to the law $X = X_0(1-\cos\omega t)$ with a velocity $v \ll c_1, c_0, c_2$. In this case the wave amplitude developed in the system is small and so also is the magnitude of the discontinuity that can be formed in the system. Suppose that phase transition is initiated at pressure $p = p_{10}$ and density $\rho = \rho_{10}$. The end of phase transition corresponds to a pressure $p = p_{20}$ and density $\rho = \rho_{20}$ (bearing in mind the initial values of the pressure p_{20} and density ρ_{20} in the pure new phase and the values of the pressure and density at the piston).

Then on the basis of the system of equations (1.1) we obtain the following equations for wave propagation in the first phase:

$$\frac{\partial\rho_1}{\partial t} + \{c_1(\rho_1) + u(\rho_1)\}\frac{\partial\rho_1}{\partial x} = 0, \tag{3.1}$$

in the phase mixture:

$$\frac{\partial\rho}{\partial t} + \{c_0(\rho) + u(\rho)\}\frac{\partial\rho}{\partial x} = \mu\frac{\partial^2\rho}{\partial x^2}, \tag{3.2}$$

and in the second phase:

$$\frac{\partial\rho_2}{\partial t} + \{c_2(\rho_2) + u(\rho_2)\}\frac{\partial\rho_2}{\partial x} = 0, \tag{3.3}$$

which are integrated for a given initial distribution $\rho_1(x, 0)$, $\rho(x, 0)$, $\rho_2(x, 0)$. Thus stated, however, the problem admits further simplifications. The time of formation of a discontinuity in the two-phase mixture due to nonlinear distortion of the wave profile is $t_0 \approx c_0/X_0\omega^2$, and the time for the phase mixture to disappear is $t_1 \sim c_2/\omega(c_2 - c_0)$, since points of the profile with pressure $p > p_{20}$ are transported with a velocity of order $c_2 > c_0$. Consequently, $t_1/t_0 \sim X_0\omega c_2/c_0(c_2 - c_0)$, and for $c_2 - c_0 \sim c_2$ we obtain $t_1/t_0 \approx u/c_0 \ll 1$. This means that the mixed-phase region disappears long before the wave profile suffers distortion in it. Inasmuch as a shock wave develops at the interface between the second phase and the phase mixture, its profile and velocity will be determined primarily by the difference $c_2 - c_0$. Here, bearing in mind that $c_2/X_0\omega^2 \approx c_1/X_0\omega^2 > c_0/X_0\omega^2$ ($c_0 < c_1, c_2$), we can neglect terms of the form $u(\rho)\partial\rho/\partial x$, $\mu\partial^2\rho/\partial x$ in (3.1)-(3.3) and treat these equations in the acoustic approximation. It is necessary, on the other hand, to take account of the nonlinear distortions in the wave as it continues to propagate after disappearance of the region of phase coexistence.

Because of the smallness of the wave amplitude, we regard it as a simple wave, i.e., assume that the entropy and appropriate Riemann invariant in it do not change [2]. The following expression holds for the transport velocity of the medium:

$$u - \int_{\rho_1}^{\rho}\frac{c(\rho)}{\rho}d\rho = u_0 - \int_{\rho_1}^{\rho_0}\frac{c(\rho)}{\rho}d\rho,$$

in which the index 0 characterizes the initial state of the system. (From here on we use the method described in [4].)

Assigning different constant values to the quantity $c(\rho)$ (c_1 in the first phase, c_0 in the two-phase region, and c_2 in the second phase), we obtain expressions for the velocity of the medium from the linearized equations (3.1)-(3.3) and the equations of continuity:

$$
\begin{gathered}
u=\frac{c_1}{\rho_0}\delta\rho \quad \text{for} \quad \rho_{10}>\rho>\rho_0, \\
u=\frac{c_0\delta\rho}{\rho_0}+\frac{(c_1-c_0)}{\rho_0}(\rho_{10}-\rho_0) \quad \text{for} \quad \rho_{20}>\rho>\rho_{10}, \\
u=\frac{c_2\delta\rho}{\rho_0}+\frac{(c_0-c_2)}{\rho_0}(\rho_{20}-\rho_0)+\frac{(c_1-c_0)}{\rho_0}(\rho_{10}-\rho_0) \quad \text{for} \quad \rho>\rho_{20}.
\end{gathered} \tag{3.4}
$$

From (3.4) we obtain for $\delta\rho$

$$
\begin{gathered}
\delta\rho=\frac{\rho_0}{c_1}u \ \text{for} \quad u<u_{10}=\frac{c_1}{\rho_0}(\rho_{10}-\rho_0), \\
\delta\rho=\frac{\rho_0}{c_0}u+\frac{(c_0-c_1)}{c_0}(\rho_{10}-\rho_0) \ \text{for} \quad u_{10}<u<u_{20}= \\
=\frac{c_0(\rho_{20}-\rho_0)}{\rho_0}+\frac{(c_1-c_0)}{\rho_0}(\rho_{10}-\rho_0), \\
\delta\rho=\frac{\rho_0}{c_2}u+\frac{(c_2-c_0)}{c_2}(\rho_{20}-\rho_0)+\frac{(c_0-c_1)}{c_2}(\rho_{10}-\rho_0) \ \text{for} \quad u>u_{20}.
\end{gathered} \tag{3.5}
$$

With phase transition taking place in the system, naturally a discontinuity (shock) is formed right at the piston. It is known [2] that a small-amplitude wave remains simple in the second approximation in this case, even in the presence of discontinuities. We therefore ignore reflection from the shock front. The equation for propagation of a simple wave generated by a piston moving according to the law $X=X_0(1-\cos wt)$ has the form

$$
x=X_0(1-\cos\omega\tau)+(t-\tau)[c(\rho)+u(\tau)], \tag{3.6}
$$

where τ is the time to formation of density ρ at the piston.

Inasmuch as $u \ll c_0$, c_1, c_2, from now on we neglect the velocity u in comparison with the sound velocity in Eq. (3.6). The upper and lower limits of the discontinuity in the shock wave belong simultaneously to the simple waves to the left and to the right of the discontinuity. Using (3.5), we write the condition for continuity of the mass flux across a discontinuity moving with velocity w relative to a fixed coordinate system:

$$
\left\{\rho_0+\frac{\rho_0u_2}{c_2}+\frac{(c_2-c_0)}{c_2}\Delta_2+\frac{(c_0-c_1)}{c_2}\Delta_1\right\}(u_2-w)=\left\{\rho_0+\frac{\rho_0u_0}{c_0}+\frac{(c_0-c_1)}{c_0}\Delta_1\right\}(u_0-w), \tag{3.7}
$$

where $\Delta_1=\rho_{10}-\rho_0$; $\Delta_2=\rho_{20}-\rho_0$; u_2 is the velocity of the medium in the second phase; and u_0 is the velocity of the medium in the two-phase mixture.

We use (3.6) to write equations describing the motion of the discontinuity:

$$
\begin{gathered}
x=X_0\{1-\cos\omega\tau_2(t)\}+[t-\tau_2(t)]c_2, \\
x=X_0\{1-\cos\omega\tau_0(t)\}+[t-\tau_0(t)]c_0,
\end{gathered} \tag{3.8}
$$

where x is the coordinate of the discontinuity.

Since the boundaries of the discontinuity vary with time, the parameters $\tau_2(t)$ and $\tau_0(t)$ characterizing the boundaries of the discontinuity are functions of the time. Differentiating (3.8) with respect to the time, we obtain, as in [4],

$$
w=(1-\tau_2')c_2,\ w=(1-\tau_0')c_0. \tag{3.9}
$$

In expressions (3.9) we have omitted the terms $\omega\tau_2'\sin\omega\tau_2$, $\omega\tau_0'\sin\omega\tau_0$ as small corrections of order u/c to the solution. After simple transformations, neglecting terms of the form u^2/c, we obtain from (3.7)

$$
\left(1-\frac{w}{c_2}\right)\sin\omega\tau_2=\left(1-\frac{w}{c_0}\right)\sin\omega\tau_0+w\frac{(c_2-c_0)}{c_2c_0}\sin\omega(t_{10}+t_{20}), \tag{3.10}
$$

where t_{10} is the time to formation of density ρ_{10} at the piston and t_{20} is the time for the density at the piston to change from ρ_{10} to ρ_{20}.

Since the discontinuity in the compression wave is formed at the interface between the second phase and the phase mixture, expression (3.10) does not contain terms of the sound velocity c_1 for the first phase.

As in [4], taking the initial conditions $\tau_2=\tau_0=t$ into account, we deduce from (3.9)

$$\tau_2 = \frac{c_0}{c_2}\tau_0 + \frac{(c_2-c_0)}{c_2}t. \tag{3.11}$$

By means of (3.9) and (3.10) we obtain

$$\sin(\omega\tau_0 + n)\sin n = n\sin\omega(t_{10}+t_{20}),$$
$$n = (\omega/2c_2)(c_2-c_0)(t-\tau_0). \tag{3.12}$$

Suppose that a pressure p_{10} is created in the medium under hydrostatic conditions. If now a compression wave is generated under these conditions, then the initial pressure of transition to the second phase is small, i.e., $\sin\omega(t_{10}+t_{20})\approx 0$. Accordingly, we deduce from (3.11) and (3.12)

$$\tau_0 = -\frac{c_2-c_0}{c_2+c_0}t,\quad \tau_2 = \frac{c_2-c_0}{c_2+c_0}t.$$

Therefore, the magnitude of the discontinuity in the compression wave is

$$\delta\rho = \rho_0\frac{X_0\omega}{c_2}\sin\omega\frac{c_2-c_0}{c_2+c_0}t.$$

The shock propagation velocity is

$$w = 2c_2c_0/(c_2+c_0).$$

The magnitude of the discontinuity increases at first, reaches a maximum at time

$$t_1 = \frac{\pi}{2\omega}\left(\frac{c_2+c_0}{c_2-c_0}\right),$$

and decreases to zero at time

$$t_2 = \frac{\pi}{\omega}\left(\frac{c_2+c_0}{c_2-c_0}\right).$$

Let us assume that the pressure p developed in the medium under hydrostatic compression satisfies the condition $p_{10} < p < p_{20}$, i.e., that the medium is situated in the two-phase coexistence region. If now a rarefaction wave $X=-X_0(1-\cos\omega t)$ is generated in the investigated system, then the reverse transition to the first phase is initiated in the medium.

In this case a rarefaction shock is formed with discontinuity of magnitude

$$\delta\rho = -\rho_0\frac{X_0\omega}{c_1}\sin\omega\frac{c_1-c_0}{c_1+c_0}t,$$

which decays to zero at time

$$t = \frac{\pi}{\omega}\left(\frac{c_1+c_0}{c_1-c_0}\right).$$

The shock propagation velocity is

$$w = 2c_1c_0/(c_1+c_0).$$

For the case in which $\sin\omega(t_{10}+t_{20})\sim 1$ in expression (3.10) it is obvious that the time of disappearance of the two-phase region is characterized by the condition $\tau_0=t_{10}$. Then, with the given approximation $\sin\omega t_{20}\approx 0$, we obtain from (3.10) subject to the condition $c_1\approx c_2$:

$$w = c_2,$$

which implies (in the given approximation) that the profile of the compression (rarefaction) wave will remain unchanged with further propagation, because the simple wave behind the discontinuity propagates with velocity c_2.

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SPHERICAL DETONATION WAVES IN MEDIA WITH VOLUMETRIC VISCOSITY

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Many solid and liquid media have volumetric viscosity appearing in dynamic processes associated with change of volume. Below we investigate detonation waves in a medium with volumetric viscosity given by the model [1] intended for the description of water-saturated soils, liquids with gas bubbles, and other multicomponent media. In these media the volume deformations are almost reversible and the tangential stresses are negligibly small, which makes it possible to investigate the effect of volumetric viscosity on the propagation characteristics of intense waves without the complicating effect of other factors. The differences in the diagrams of the corresponding shock (dynamic) compression and equilibrium state (static compression), and also the time required for establishing equilibrium in these media, are small. In the present work the problem of propagation of a spherical wave generated by the detonation of an explosive charge in a medium with volumetric viscosity and also for a nonviscous medium with the compressibility diagram corresponding to the equilibrium state is solved with the use of a computer. The corresponding results for plane waves were obtained in [1-3]. In the case of spherical waves in unsaturated soils it is necessary to use the Mises–Schleicher plasticity condition [4]. Models where the viscosity term is introduced in the plasticity condition [5] are also recommended for describing dynamic processes in solids.

§ 1. We consider waves in a water-saturated soil, i.e., a three-dimensional medium (solid particles, water, gas bubbles) described by the model of [1]. We denote by α_1, α_2, and α_3 the volume content of the gaseous, liquid, and solid components, by V_{10}, V_{20}, and V_{30} their volumes, by ρ_{10}, ρ_{20}, and ρ_{30} their densities, by c_{10}, c_{20}, and c_{30} the speed of sound in them, by ρ_0 the density of the three-dimensional medium, and by V_0 its specific volume. All the quantities pertain to the atmospheric pressure p_0, $\rho_0 = \alpha_1\rho_{10} + \alpha_2\rho_{20} + \alpha_3\rho_{30}$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

At a pressure p these parameters are denoted by V_1, V_2, V_3, ρ_1, ρ_2, ρ_3, ρ, and V, respectively. In water with gas bubbles $\alpha_3 = 0$.

It is assumed that in the free state all the components are compressed according to the equation

$$p = p_0 + \frac{\rho_{i0}c_{i0}^2}{\gamma_i}\left[\left(\frac{V_{i0}}{V_i}\right)^{\gamma_i} - 1\right] \tag{1.1}$$

(i is the number of the component) which corresponds to the Poisson adiabat for a gas and the theta equation for the liquid and solid components.

The gas in the medium occurs in the form of small-scale bubbles isolated from each other by the remaining components. Under the action of a load the liquid and solid components are compressed instantaneously, while the gaseous component gets compressed in a finite time, since its compression is caused by the displacement of the other components and by the filling of the initial volume of the bubbles by the other components. Therefore, the compression of air in the medium is given by the equation

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